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A Note on the Tangential Triangle

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Abstract. Given an obtuse triangle ABC. We find the distance from the symmedian point of triangle ABC to the Gergonne point of its tangential triangle.

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Given triangle ABC with side lengths $BC = a, CA = b$ and $AB = c$. If triangle ABC is acute, then the distance from the symmedian point of triangle ABC to the Gergonne point of its tangential triangle T is equal to 0 (See e.g. Weisstein [2]).

Theorem 1. *If triangle ABC is obtuse such that $c > a$ and $c > b$, then the distance d from the symmedian point of triangle ABC to the Gergonne point of its tangential triangle T is*

$$d = \frac{8a^2b^2c^3\sqrt{E_1}}{(b^2+c^2-a^2)(c^2+a^2-b^2)(a^2+b^2+c^2)E_2}$$

where

$E_1 = -12a^6c^2b^4 + 12a^6c^4b^2 + 18a^8b^2c^2 + 45a^8b^4 + a^8c^4 - 60a^6b^6 + 4a^6c^6 - 18a^{10}b^2 - 6a^{10}c^2 - 3a^4c^8 + 45a^4b^8 - 18a^2b^{10} + 2a^2c^{10} - 6b^{10}c^2 - 6b^{10}c^2 - 3b^4c^8 + 2b^2c^{10} + b^8c^4 + 4b^6c^6 + 3a^{12} + 3b^{12} - c^{12} - 12a^4b^6c^2 - 4a^4c^6b^2 + 18a^2b^8c^2 + 6a^2b^2c^8 - 4a^2b^4c^6 + 12b^6a^2c^4 - 26b^4a^4c^4$

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and

$$E_2 = a^6 + 6b^2a^2c^2 + b^6 - b^2a^4 - b^4a^2 - 3c^2a^4 + 3c^4a^2 - 3c^2b^4 + 3c^4b^2 - c^6.$$

Proof. We use barycentric coordinates. We refer the reader to [1].

The barycentric coordinates of the symmedian point K of triangle ABC are as follows: $K = (a^2, b^2, c^2)$.

The tangential triangle T of triangle ABC is the anticevian triangle of the symmedian point K, so that the barycentric coordinates of $T = T_AT_BT_C$ are as follows (see [3], page 54):

$$T_A = (-a^2, b^2, c^2), \quad T_B = (a^2, -b^2, c^2), \quad T_C = (a^2, b^2, -c^2).$$

The side lengths of triangle T are as follows (see [2], formulas (2)-(4)):

$$a_T = \frac{2a^3bc}{|a^4 - (b^2 - c^2)^2|}, \quad b_T = \frac{2ab^3c}{|b^4 - (c^2 - a^2)^2|}, \quad c_T = \frac{2abc^3}{|c^4 - (a^2 - b^2)^2|}.$$

Since $c^2 > a^2 + b^2$, we take the following positive values of the side lengths:

$$a_T = \frac{2a^3bc}{(a^2 - b^2 + c^2)(c^2 - a^2 - b^2)},$$
$$b_T = \frac{2ab^3c}{(b^2 - a^2 + c^2)(c^2 - a^2 - b^2)},$$
$$c_T = \frac{2abc^3}{(b^2 - a^2 + c^2)(a^2 - b^2 + c^2)}.$$

Then the barycentric coordinates of the Gergonne point $Ge_T = (uGe_T, vGe_T, wGe_T)$ wrt triangle T are as follows:

$$uGe_T = (a_T - b_T + c_T)(a_T + b_T - c_T),$$
$$vGe_T = (b_T - c_T + a_T)(b_T + c_T - a_T),$$
$$wGe_T = (c_T - a_T + b_T)(c_T + a_T - b_T).$$

or equivalently,

$$uGe_T = -(b + c - a)(c + a - b)(a + b - c)(a + b + c)(a^2 - b^2 + c^2),$$
$$vGe_T = -(b + c - a)(c + a - b)(a + b - c)(a + b + c)(b^2 + c^2 - a^2),$$
$$wGe_T = (b^2 + c^2 - a^2)(c^2 + a^2 - b^2)(a^2 + b^2 - c^2).$$

Now, by using formula (10) in [1], we obtain the barycentric coordinates of the Gergonne point of triangle T wrt triangle ABC as follows:

$$uGe_T = a^2(-4a^6b^2 + 12b^2a^4c^2 - 12b^4a^2c^2 + 4b^2a^2c^4 + 4b^6c^2 - 4c^6a^2 + c^8 + a^8 + b^8 + 6b^4a^4 - 4b^6a^2 - 4c^2a^6 + 6c^4a^4 - 10c^4b^4 + 4c^6b^2),$$
$$vGe_T = b^2(-4a^6b^2 - 12b^2a^4c^2 + 12b^4a^2c^2 + 4b^2a^2c^4 - 4b^6c^2 + 4c^6a^2 + c^8 + a^8 + b^8 + 6b^4a^4 - 4b^6a^2 + 4c^2a^6 - 10c^4a^4 + 6c^4b^4 - 4c^6b^2),$$
$$wGe_T = c^2(b^4 - 2b^2a^2 + a^4 - 4abc^2 - 2c^2a^2 - 2c^2b^2 + c^4)(b^4 - 2b^2a^2 + a^4 + 4abc^2 - 2c^2a^2 - 2c^2b^2 + c^4).$$

Finally, by using formula (9) in [1] we find the distance d between points Ge_T and K. □

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