International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 3, 2018, pp.104-109.
Received 20 May 2018. Published on-line 11 July 2018
web: http://www.journal-1.eu/

An Ellipse Through 12 Points and Golden Triangle

Dao Thanh Oai
Kien Xuong, Thai Binh, Vet Nam
e-mail: daothanhoai@hotmail.com

Abstract. We introduce an ellipse through 12 points and a triangle we call "*Golden Ellipse*" and "*Golden Triangle*" in a reference triangle and its variant.

Keywords. Conic section, Ellipse, Golden ratio.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. An ellipse through 12 points and golden triangle

Theorem 1.1 ([1], [2]). *Let ABC be a triangle, let points B_a, C_a be chosen on BC, points C_b, A_b be chosen on CA, points A_c, B_c be chosen on AB, such that B_cC_b, A_cC_a, A_bB_a parallel to BC, CA, AB respectively. Let $A' = A_cC_a \cap A_bB_a$ define B', C' cycllically. Let $A'' = BC_b \cap CB_c$, $B'' = CA_c \cap AC_a$, $C'' = AB_a \cap BA_b$. Then three statements as follows are equivalent:*

1. Points $A'' \equiv A'$ and $B'' \equiv B'$ and $C'' \equiv C'$.

2. 12 points: B_a, C_a, C_b, A_b, A_c, B_c and midpoints of AB', AC', BC', BA', CA', CB' lie on an ellipse.

3.
$$\frac{\overline{BC}}{\overline{BC_a}} = \frac{\overline{CB}}{\overline{CB_a}} = \frac{\overline{CA}}{\overline{CA_b}} = \frac{\overline{AC}}{\overline{AC_b}} = \frac{\overline{AB}}{\overline{AB_c}} = \frac{\overline{BA}}{\overline{BA_c}} = \frac{\sqrt{5}+1}{2}$$

2. Some variant of Golden triangle

Theorem 2.1 ([1]). *Let ABC be a triangle, let points B_a, C_a be chosen on segment BC, points C_b, A_b be chosen on segment CA, points A_c, B_c be chosen on segment AB, such that:*
$$\frac{\overline{BC}}{\overline{BC_a}} = \frac{\overline{CB}}{\overline{CB_a}} = \frac{\overline{CA}}{\overline{CA_b}} = \frac{\overline{AC}}{\overline{AC_b}} = \frac{\overline{AB}}{\overline{AB_c}} = \frac{\overline{BA}}{\overline{BA_c}} = t$$
Where $(0 < t < 2)$. Let $BC_b \cap CB_c = A'$ define B', C' cycllically.

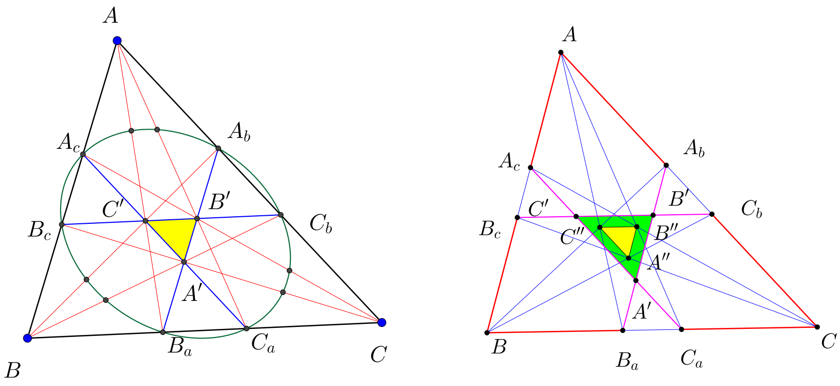

Figure 1. Theorem 1.1

1. The Euler lines of two triangle ABC and $A'B'C'$ are concides.

2. Let H, O be the circumcenter and orthocenter of ABC respectively and H', O' be the circumcenter and orthocenter of $A'B'C'$ respectively, then: $\frac{HO'}{O'O} = \frac{OH'}{H'O'}$ iff $t = \phi = \frac{\sqrt{5}+1}{2}$. In this case $\frac{HO'}{O'O} = \frac{OH'}{H'O'} = \phi = \frac{\sqrt{5}+1}{2}$ and $\frac{S_{ABC}}{S_{A'B'C'}} = \phi^8$

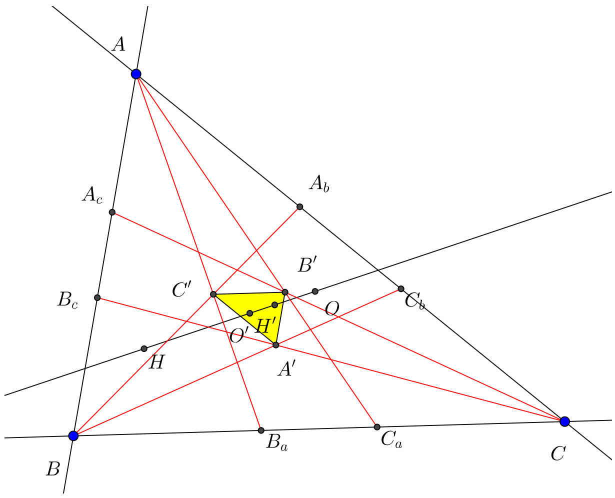

Figure 2. Theorem 2.1

Theorem 2.2. *The Golden triangle $A'B'C'$ perspective to arbitrary Kiepert triangle*

Theorem 2.3 (Second Golden triangle)**.** *Let ABC be a triangle, let points B_a, C_a be chosen on segment BC, points C_b, A_b be chosen on segment CA, points A_c, B_c be chosen on segment AB, such that:*

$$\frac{\overline{BC}}{\overline{BC_a}} = \frac{\overline{CB}}{\overline{CB_a}} = \frac{\overline{CA}}{\overline{CA_b}} = \frac{\overline{AC}}{\overline{AC_b}} = \frac{\overline{AB}}{\overline{AB_c}} = \frac{\overline{BA}}{\overline{BA_c}} = t$$

Let $BA_b \cap CA_c = A'$ define B', C' cycllically.

1. The Euler lines of two triangle ABC and $A'B'C'$ are concides.

2. Let O be the circumcenter of ABC respectively and H', O' be the circumcenter and orthocenter of $A'B'C'$ respectively, then: $\frac{OO'}{O'H'} = t$ iff $t = \phi = \frac{\sqrt{5}+1}{2}$. In this case $\frac{S_{ABC}}{S_{A'B'C'}} = 5\phi^4$

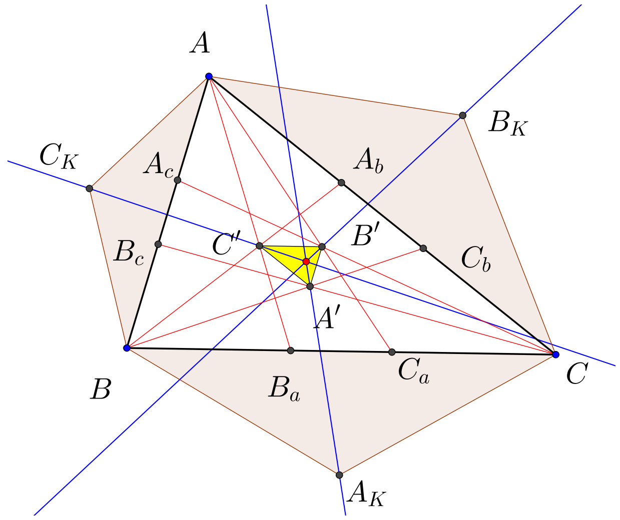

FIGURE 3. Theorem 2.2

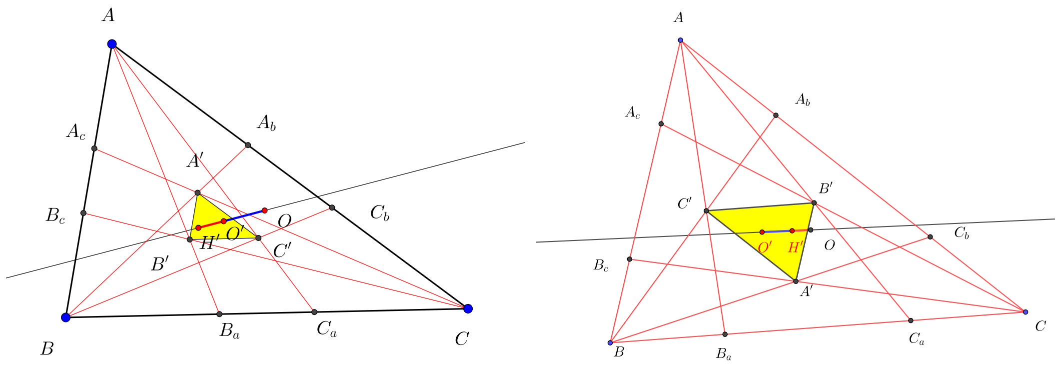

FIGURE 4. Theorem 2.3 and Theorem 2.4

Theorem 2.4 (Third Golden triangle). *Let ABC be a triangle, let points B_a, C_a be chosen on segment BC, points C_b, A_b be chosen on segment CA, points A_c, B_c be chosen on segment AB, such that:*

$$\frac{\overline{B_aC_a}}{\overline{BB_a}} = \frac{\overline{C_aB_a}}{\overline{CC_a}} = \frac{\overline{C_bA_b}}{\overline{CC_b}} = \frac{\overline{A_bC_b}}{\overline{AA_b}} = \frac{\overline{A_cB_c}}{\overline{AA_c}} = \frac{\overline{B_cA_c}}{\overline{BB_c}} = t$$

Let $BC_b \cap CB_c = A'$ define B', C' cycllically.

1. The Euler lines of two triangle ABC and $A'B'C'$ are concides.

2. Let O be the circumcenter of ABC respectively and H', O' be the circumcenter and orthocenter of $A'B'C'$ respectively, then:$\frac{O'H'}{H'O} = t$ iff $t = \phi = \frac{\sqrt{5}+1}{2}$. In this case $\frac{S_{ABC}}{S_{A'B'C'}} = 3\phi + 10$

Theorem 2.5 (Fourth Golden triangle). *Let ABC be a triangle, let points B_a, C_a be chosen on segment BC, points C_b, A_b be chosen on segment CA, points A_c, B_c be chosen on segment AB, such that:*

$$\frac{\overline{B_aC_a}}{\overline{BB_a}} = \frac{\overline{C_aB_a}}{\overline{CC_a}} = \frac{\overline{C_bA_b}}{\overline{CC_b}} = \frac{\overline{A_bC_b}}{\overline{AA_b}} = \frac{\overline{A_cB_c}}{\overline{AA_c}} = \frac{\overline{B_cA_c}}{\overline{BB_c}} = t$$

Let $BC_b \cap CB_c = A'$ *define* B', C' *cycllically.*

1. The Euler lines of two triangle ABC *and* $A'B'C'$ *are concides.*

2. Let O *be the circumcenter of* ABC *respectively and* H', O' *be the circumcenter and orthocenter of* $A'B'C'$ *respectively, then:* $\frac{H'O'}{O'O} = t$ *iff* $t = \phi = \frac{\sqrt{5}+1}{2}$*. In this case* $\frac{S_{ABC}}{S_{A'B'C'}} = 10 - \frac{3}{\phi}$

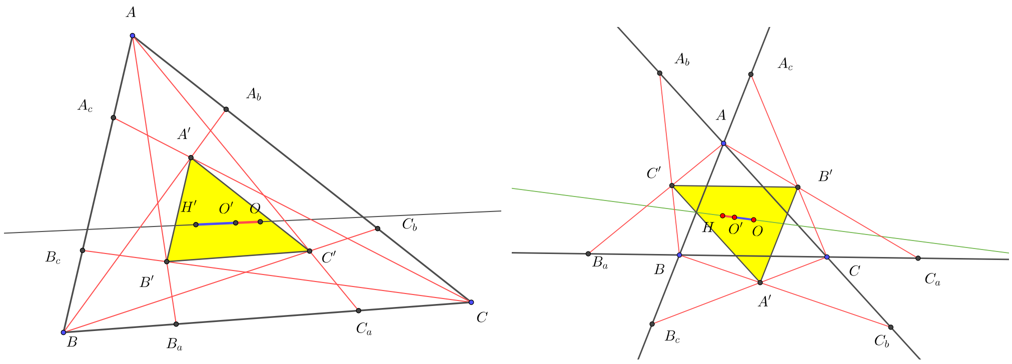

Figure 5. Theorem 2.5 and Theorem 2.6

Theorem 2.6 (Fiveth Golden triangle)**.** *Let* ABC *be a triangle, let points* B_a, C_a *be chosen on line* BC *but not in segment* BC*; points* C_b, A_b *be chosen on line* CA *but not in segment* CA*; points* A_c, B_c *be chosen on line* AB *but not in segment* AB*, such that:*

$$\frac{\overline{BC}}{\overline{B_aB}} = \frac{\overline{CB}}{\overline{C_aC}} = \frac{\overline{CA}}{\overline{C_bC}} = \frac{\overline{AC}}{\overline{A_bA}} = \frac{\overline{AB}}{\overline{A_cA}} = \frac{\overline{BA}}{\overline{B_cB}} = t$$

Let $BC_b \cap CB_c = A'$ *define* B', C' *cycllically.*

1. The Euler lines of two triangle ABC *and* $A'B'C'$ *are concides.*

2. Let H, O *be the circumcenter and orthocenter of* ABC *respectively and* O' *be the circumcenter of* $A'B'C'$*, then:* $\frac{OO'}{O'H} = t$ *iff* $t = \phi = \frac{\sqrt{5}+1}{2}$*. In this case* $\frac{S_{ABC}}{S_{A'B'C'}} = \frac{\phi^4}{5}$

Theorem 2.7 (Sixth Golden triangle)**.** *Let* ABC *be a triangle, let points* B_a, C_a *be chosen on line* BC *but not in segment* BC*; points* C_b, A_b *be chosen on line* CA *but not in segment* CA*; points* A_c, B_c *be chosen on line* AB *but not in segment* AB*. such that:*

$$\frac{\overline{BC}}{\overline{B_aB}} = \frac{\overline{CB}}{\overline{C_aC}} = \frac{\overline{CA}}{\overline{C_bC}} = \frac{\overline{AC}}{\overline{A_bA}} = \frac{\overline{AB}}{\overline{A_cA}} = \frac{\overline{BA}}{\overline{B_cB}} = t$$

Let $BA_b \cap CA_c = A'$ *define* B', C' *cycllically.*

1. The Euler lines of two triangle ABC *and* $A'B'C'$ *are concides.*

2. Let H, O *be the circumcenter and orthocenter of* ABC *respectively and* O' *be the circumcenter of* $A'B'C'$*, then:* $\frac{OO'}{OH} = t$ *iff* $t = \phi = \frac{\sqrt{5}+1}{2}$*. In this case* $\frac{S_{ABC}}{S_{A'B'C'}} = \frac{1}{5\phi^4}$

Theorem 2.8 (Seventh Golden triangle)**.** *Let* ABC *be a triangle, let points* B_a, C_a *be chosen on line* BC *but not in segment* BC*; points* C_b, A_b *be chosen on line* CA

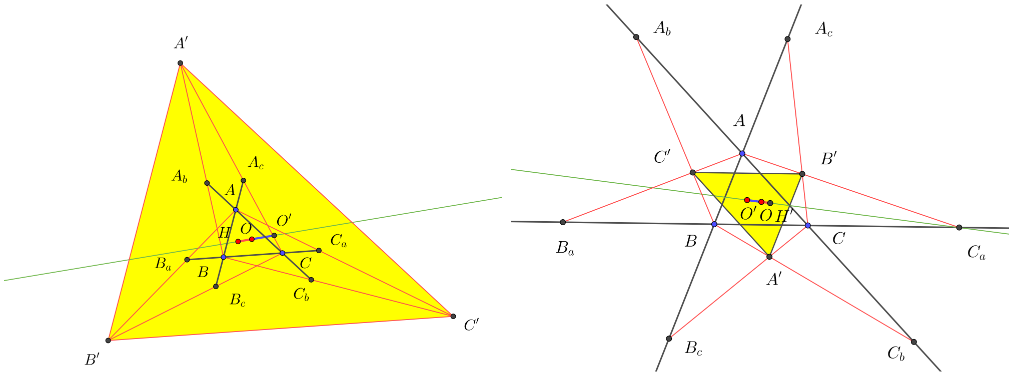

FIGURE 6. Theorem 2.7 and Theorem 2.8

but not in segment CA; points A_c, B_c be chosen on line AB but not in segment AB, such that:

$$\frac{\overline{B_aB}}{\overline{BC}} = \frac{\overline{C_aC}}{\overline{CB}} = \frac{\overline{C_bC}}{\overline{CA}} = \frac{\overline{A_bA}}{\overline{AC}} = \frac{\overline{A_cA}}{\overline{AB}} = \frac{\overline{B_cB}}{\overline{BA}} = t$$

Let $BC_b \cap CB_c = A'$ define B', C' cycllically.

1. The Euler lines of two triangle ABC and $A'B'C'$ are concides.

2. Let H, O be the circumcenter and orthocenter of ABC respectively and O' be the circumcenter of $A'B'C'$, then: $\frac{O'O}{OH'} = t$ iff $t = \phi = \frac{\sqrt{5}+1}{2}$. In this case $\frac{S_{ABC}}{S_{A'B'C'}} = \frac{5}{\phi^4}$

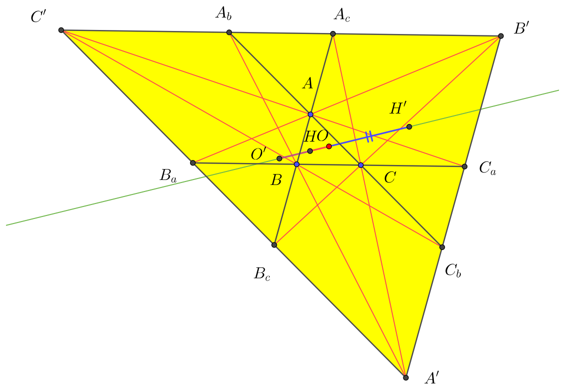

FIGURE 7. Theorem 2.9

Theorem 2.9 (Eighth Golden triangle)**.** *Let ABC be a triangle, let points B_a, C_a be chosen on line BC but not in segment BC; points C_b, A_b be chosen on line CA but not in segment CA; points A_c, B_c be chosen on line AB but not in segment AB, such that:*

$$\frac{\overline{B_aB}}{\overline{BC}} = \frac{\overline{C_aC}}{\overline{CB}} = \frac{\overline{C_bC}}{\overline{CA}} = \frac{\overline{A_bA}}{\overline{AC}} = \frac{\overline{A_cA}}{\overline{AB}} = \frac{\overline{B_cB}}{\overline{BA}} = t$$

Let $BA_b \cap CA_c = A'$ define B', C' cycllically.

1. The Euler lines of two triangle ABC and $A'B'C'$ are concides.

2. Let H, O *be the circumcenter and the orthocenter of* ABC *respectively;* O', H' *be the circumcenter and the orthocenter of* $A'B'C'$ *respectively, then:* $\frac{H'O}{OO'} = \frac{O'H}{HO}$ *iff* $t = \phi = \frac{\sqrt{5}+1}{2}$*. In this case* $\frac{H'O}{OO'} = \frac{O'H}{HO} = \phi = \frac{\sqrt{5}+1}{2}$ *and* $\frac{S_{ABC}}{S_{A'B'C'}} = \frac{1}{\phi^8}$

REFERENCES

[1] Dao Thanh Oai, Problem 12050 and Solutions, *The American Mathematical Monthly*, 125:6, 562-570, DOI: 10.1080/00029890.2018.1460990.

[2] An ellipse through 12 points related to Golden ratio, *mathoverflow*, available at `https://mathoverflow.net/questions/303501`