International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 3, 2018, pp.88-96.
Received 15 May 2018. Published on-line 1 July 2018
web: http://www.journal-1.eu/

Some Equilateral Triangles Perspective to the Reference Triangle ABC

Dao Thanh Oai
Kien Xuong, Thai Binh, Vet Nam
e-mail: daothanhoai@hotmail.com

Abstract. We give 25 new equilateral triangles perspective to the reference triangle. These equilateral triangle associated with some special points and some hyperbola, for example Fermat points, Isodynamic points, Symmedian point, Morley centers, isogonal conjugate point, Kiepert hyperbola, Morley hyperbola.

Keywords. Euclidean Geometry, equilateral triangle, perspective triangle, perspector, Kiepert hyperbola, Morley hyperbola, Morley triangle, Napoleon triangle.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Some Equilateral Triangles Perspective to Triangle ABC Associated with the Fermat points and Kiepert hyperbola

Theorem 1.1. *Let ABC be a triangle, F be the fist Fermat point. Circle (FBC) meets AC, AB again at A_b, A_c. Define B_c, B_a, C_a, C_b cyclically. Let A', B', C' be centers of three circles (AB_cC_b), (BC_aA_c), (CA_bB_a) respectively. Then triangle $A'B'C'$ is equilateral and perspective to ABC, the perspector is $X(4)$. ABC orthologic to $A'B'C'$ the orthologic center is $X(13)$. Center of $A'B'C'$ is $X(381)$. $A'B'C'$ is reflection of the outer Napoleon triangle in $X(5)$.*

Theorem 1.2. *Let ABC be a triangle, F be the second Fermat point. Circle (FBC) meets AC, AB again at A_b, A_c respectively. Define B_c, B_a, C_a, C_b cyclically. Let A', B', C' be centers of three circles (AB_cC_b), (BC_aA_c), (CA_bB_a) respectively. Then triangle $A'B'C'$ is equilateral and perspective to ABC, the perspector is $X(4)$. ABC orthologic to $A'B'C'$ the orthologic center is $X(13)$. Center of $A'B'C'$ is $X(381)$. $A'B'C'$ is reflection of the inner Napoleon triangle in $X(5)$.*

You can see more above Theorem 1.1 and Theorem 1.2 in [1].

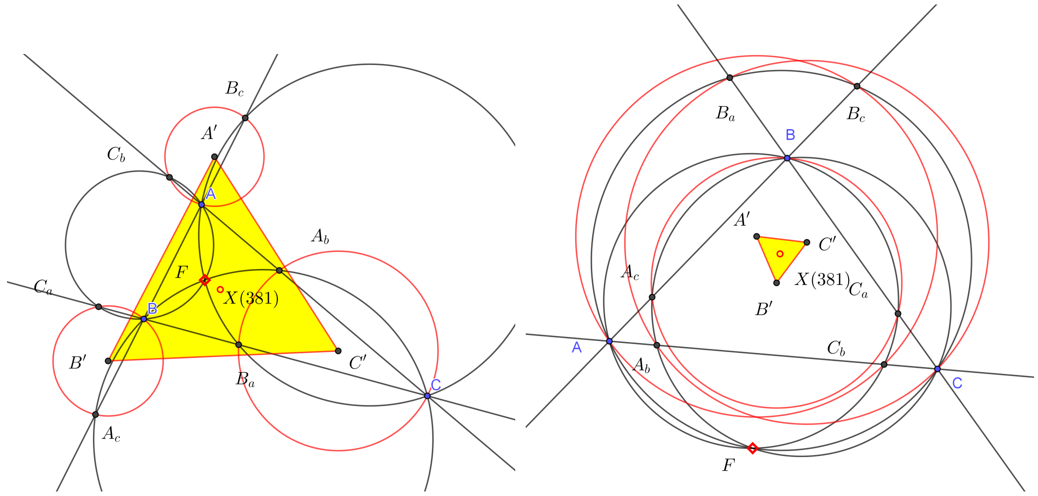

Figure 1. Theorem 1.1 and Theorem 1.2

Theorem 1.3. *Let ABC be a triangle, F be the fist Fermat point. Circle (FBC) meets AC, AB again at A_b, A_c respectively. Define B_c, B_a, C_a, C_b cyclically. Let A', B', C' be centroids of three triangles AB_cC_b, BC_aA_c, CA_bB_a respectively. Then triangle $A'B'C'$ is equilateral and perspective to ABC, the perspector is $X(6)$.*

Theorem 1.4. *Let ABC be a triangle, F be the second Fermat point. Circle (FBC) meets AC, AB again at A_b, A_c. Define B_c, B_a, C_a, C_b cyclically. Let A', B', C' be centroids of three triangles AB_cC_b, BCaAc, CAbBa respectively. Then triangle $A'B'C'$ is equilateral and perspective to ABC, the perspector is $X(6)$.*

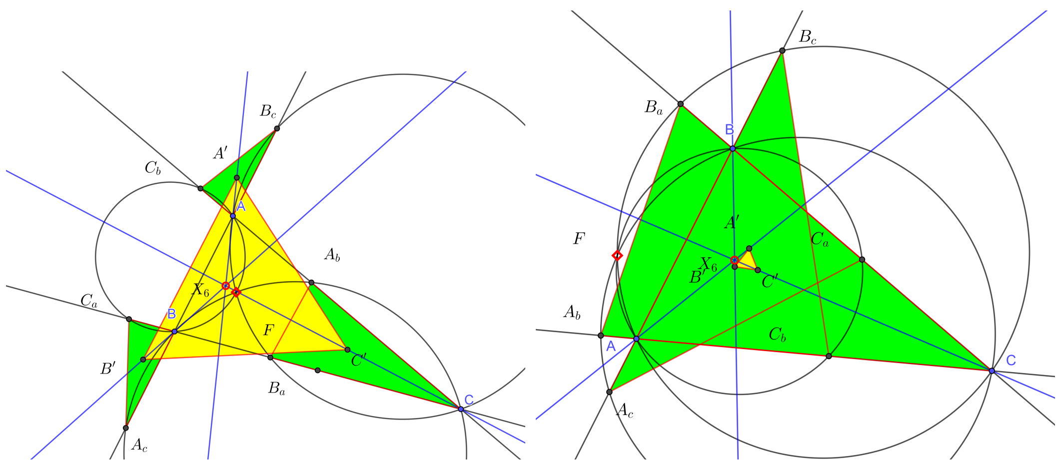

Figure 2. Theorem 1.3 and Theorem 1.4

You can see more above Theorem 1.3 and Theorem 1.4 in [2].

Theorem 1.5. *Let ABC be a triangle and $A'B'C'$ is outer (or inner) Fermat triangle, Let t be a real number with $t \geq 0$. Let L_a be the radical line of two circles (A, t) and (A', BC). Define L_b, L_c cyclically. Let $A'' = L_b \cap L_c$; define B'', C'' cyclically. Then triangle $A''B''C''$ is equilateral and perspetive to ABC, the perspector lies on the Kiepert hyperbola.*

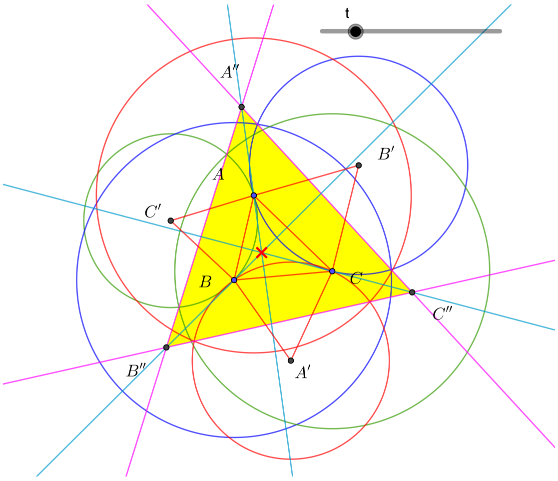

FIGURE 3. Theorem 1.5

Theorem 1.6. *Let ABC be a triangle and $A'B'C'$ is outer (or inner) Fermat triangle. Let L_a be the polar line of A respect to (A', BC). Define L_b, L_c cyclically. Let $A'' = L_b \cap L_c$. Define B'', C'' cyclically. Then triangle $A''B''C''$ is equilateral and perspetive to ABC, the perspector lies on the Kiepert hyperbola.*

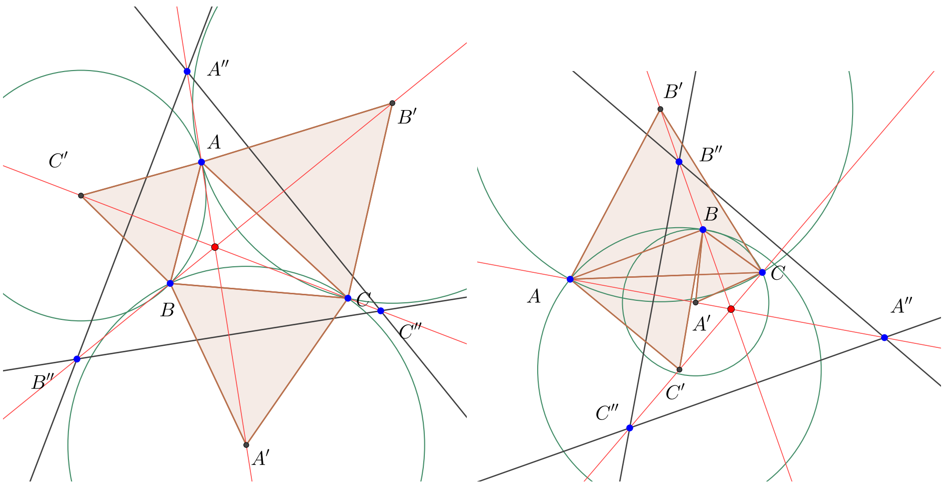

FIGURE 4. Theorem 1.6

You can see more above Theorem 1.6 in [3].

Theorem 1.7 ([9])**.** *Let ABC be a triangle, $A'B'C'$ is the inner (or outer) Fermat triangle, P be a point in the plane. Let A'', B'', C'' are reflection of A', B', C' in P respectively. Let A_0, B_0, C_0 are the midpoints of AA'', BB'', CC''. Then $A_0B_0C_0$ is an equilateral triangle. The side length A_0B_0 is independent of P.*

Theorem 1.8 ([9])**.** *Let ABC be a triangle, $A'B'C'$ is the inner (or outer) Fermat triangle, P be a point in the plane. Let A'', B'', C'' are reflection of A, B, C in P respectively. Let A_0, B_0, C_0 are the midpoints of $A'A''$, $B'B''$, $C'C''$. Then $A_0B_0C_0$ is an equilateral triangle. The side length A_0B_0 is independent of P.*

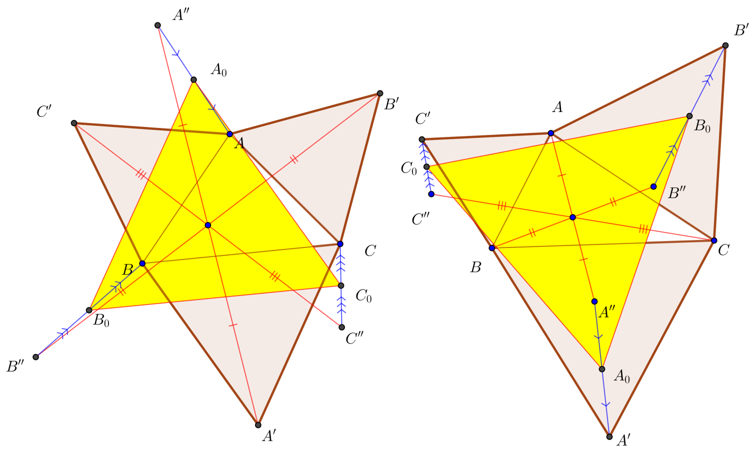

Figure 5. Theorem 1.7 and Theorem 1.8

You can see more properties of inner and outer Fermat triangle in [10].

Theorem 1.9. *Let ABC be a triangle, F be the first (or second) Fermat point and A', B', C' be three points on the Fermat lines AF, BF, CF respectively so that $AA' = BB' = CC'$. Let A_0, B_0, C_0 be the centroids of $\triangle BCA'$, $\triangle CAB'$ $\triangle ABC'$ respectively, then*

1. *$A_0B_0C_0$ is an equilateral triangle*
2. *Two triangles $A_0B_0C_0$, ABC are perspective.*

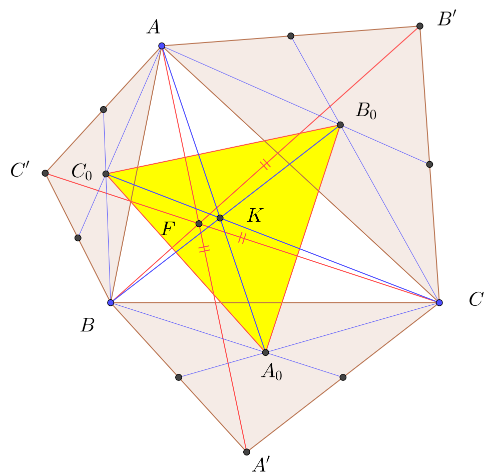

Figure 6. Theorem 1.9

You can see more properties in the configuration of Theorem 1.9 in [11].

2. Some Equilateral Triangles Perspective to ABC Associated with the isodynamic points

Theorem 2.1. *Let ABC be a triangle, Let P be the first (or the second) isodynamic point. The tangent line of (BPC) at P meets circles (APB), (APC) again at A_c, A_b respectively. Define B_c, B_a, C_a, C_b cyclically. Let CB_c meets BC_b at A', define B', C' cyclically, then $A'B'C'$ is an equilateral triangle.*

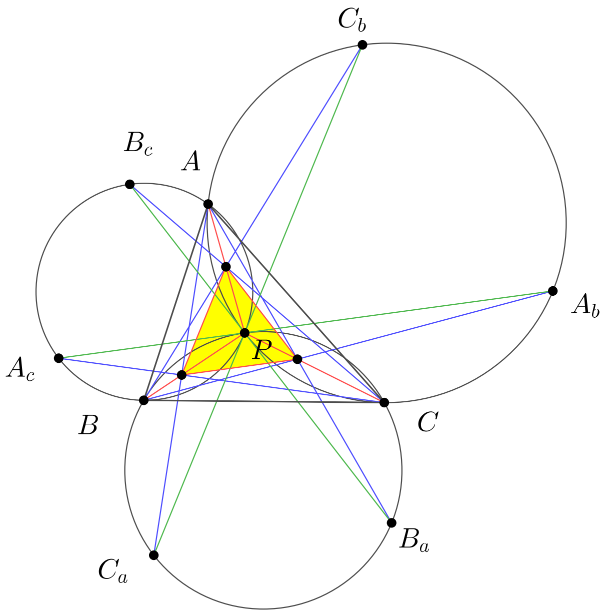

FIGURE 7. Theorem 2.1

Theorem 2.2. *Let ABC be a triangle, P is the firt (or the second) Isodynamic point. The circle (B, BP) meets the circle (C, CP) again at A', define B', C' cyclically, then $A'B'C'$ form an equilateral triangle with center is the first (or second) Fermat point.*

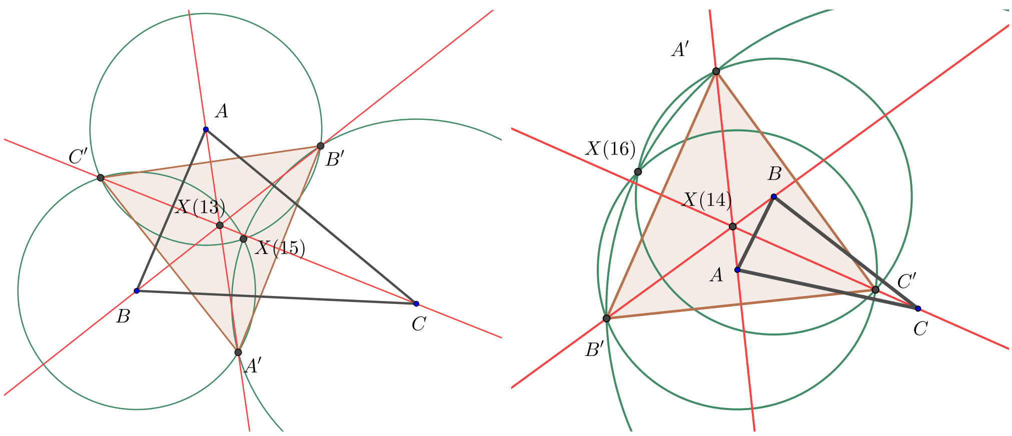

FIGURE 8. Theorem 2.2

You can see more above Theorem 2.2 in [4]

3. Some Equilateral Triangles Perspective to ABC Associated with the symmedians point

Theorem 3.1. *Let ABC be a triangle, points B_a, C_a be chosen on BC, points C_b, A_b be chosen on CA, points A_c, B_C be chosen on AB such that B_aA_b, C_aA_c, C_bB_c through the symmedian point and parallel to BC, CA, AB respectively. Construct equilateral $A_bA'A_c$, $B_cB'B_a$, $C_aC'C_b$ all outwards. Then triangle $A'B'C'$ is equilateral and perspective to ABC. The perspector is the second Fermat point. The circumcircle $(A'B'C')$ tangent to the inner Napoleon triangle at the second Fermat point. The orthologic center of $A'B'C'$ to ABC is the Symmedian point.*

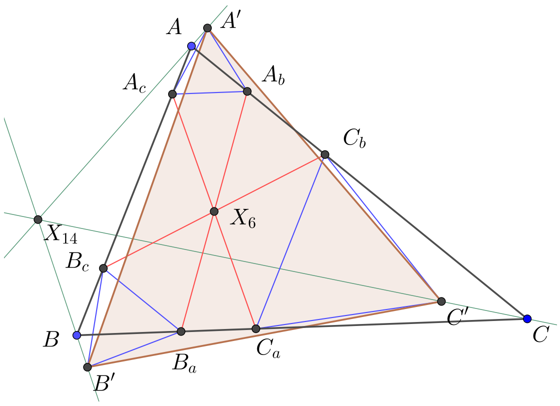

Figure 9. Theorem 4

Theorem 3.2. *Let ABC be a triangle, points B_a, C_a be chosen on BC, points C_b, A_b be chosen on CA, points A_c, B_C be chosen on AB such that B_aA_b, C_aA_c, C_bB_c through the symmedian point and parallel to BC, CA, AB respectively. Construct equilateral $A_bA'A_c$, $B_cB'B_a$, $C_aC'C_b$ all inwards. Then triangle $A'B'C'$ is equilateral and perspective to ABC. The perspector is the first Fermat point. The circumcircle $(A'B'C')$ tangent to the inner Napoleon triangle at the first Fermat point. The orthologic center of $A'B'C'$ to ABC is the Symmedian point.*

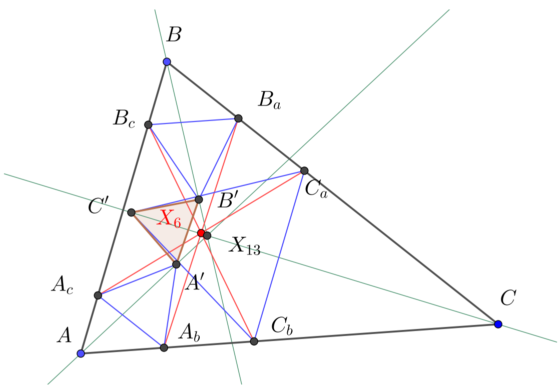

Figure 10. Theorem 4

You can see more above Theorem 3.1 and Theorem 3.2 in [5]

4. SOME EQUILATERAL TRIANGLES PERSPECTIVE TO ABC ASSOCIATED WITH THE MORLEY CENTERS AND MORLEY HYPERBOLA

Theorem 4.1 ([6])**.** *Let ABC be a triangle, $A'B'C'$ be the orthic triangle. Let BC meets $B'C'$ at A_0. Define B_0, C_0 cyclically. The angle trisection of $\angle(BC, B'C')$ meets AB, AC at A_c, A_b respectively Such that $\angle(A_bA_c, BC) = \frac{2}{3}\angle(A_bA_c, B'C')$. Define B_c, B_a, C_a, C_b cyclically.*

1. Then six points A_b, A_c, B_c, B_a, C_a, C_b lie on a circle.

2. Let $A''B''C''$ be the triangle fomed by A_bA_c, B_cB_a, C_aC_b. Then $A''B''C''$ is equilateral and persepctive to ABC and orthic triangle.

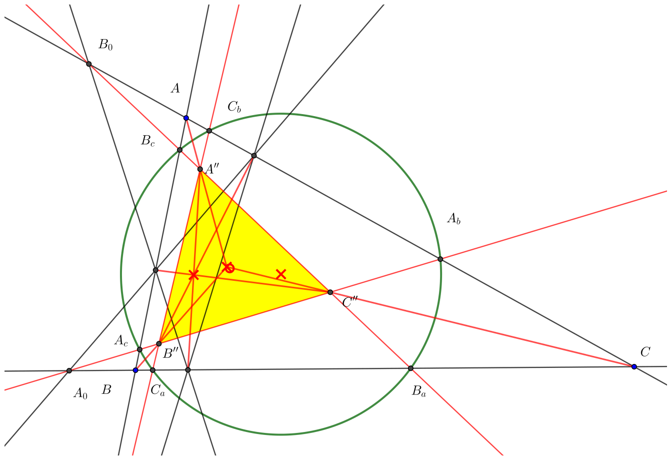

FIGURE 11. Theorem 4.1

Theorem 4.2. *Let ABC be a triangle and $A'B'C'$ be a Morley triangle of triangle ABC. Let A'', B'', C'' be the centers of three circles (BCA'), (CAB'), (BAC'). Then the centroid of Morley triangle is the Fermat point of triangle $A''B''C''$*

Theorem 4.3. *Let ABC be a triangle, let $A'B'C'$ be the first Morley triangle (or the second Morley triangle, or the third Morley trianhle). Let B_a, C_a on BC such that $A'B_aC_a$ be an equilateral triangle. Define C_b, A_b, A_c, B_c cyclically. Let A'', B'', C'' be the midpoints of A_bA_c, B_cB_a, C_aC_b respectively. Then triangle $A''B''C''$ is equilateral triangle and perspective to ABC. $A''B''C''$ homothetic to the Morley triangle.*

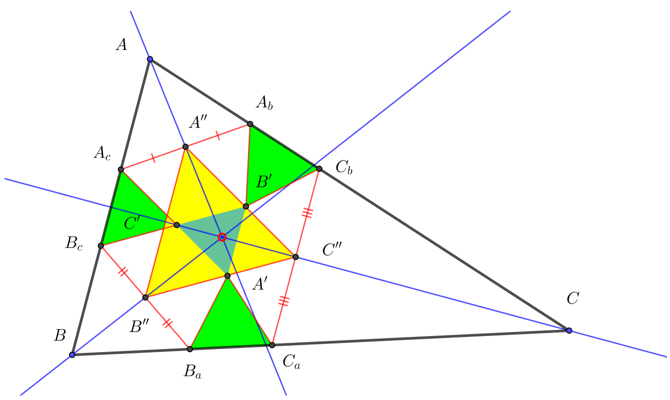

FIGURE 12. Theorem 4.3

You can see more above Theorem 4.3 in [7] and the Morley hyperbola in [8].

5. Some Equilateral Triangles Perspective to ABC Associated with the isogonal conjugate points

Theorem 5.1. *Let P be a point in the plane of a triangle ABC. Let B_a and C_a be points on BC such that the triangle PB_aC_a is equilateral. Define the pairs C_b, A_b and A_b, B_c cyclically. A' be the point such that $A'A_bA_c$ be an equilateral triangle and $A'A_bA_c$ the same orientation than ABC. Define B', C' cyclically. Then triangle $A'B'C'$ is equilateral and perspective to ABC, the perspector is isogonal conjugate of P.*

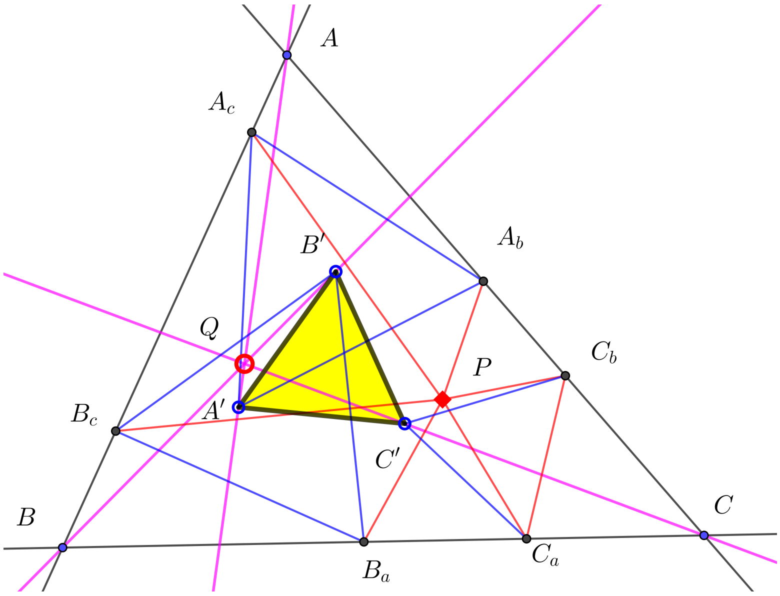

FIGURE 13. Theorem 5.1

References

[1] X(16626) = PERSPECTOR OF THESE TRIANGLES: 11th FERMAT-DAO AND JOHNSON, *Encyclopedia of Triangle Centers,* available at `http://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16626`

[2] X(16636) = PERSPECTOR OF THESE TRIANGLES: 13th FERMAT-DAO AND 1st FERMAT-DAO, *Encyclopedia of Triangle Centers,* available at `http://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16636`

[3] X(16960) = HOMOTHETIC CENTER OF THESE TRIANGLES: 15th FERMAT-DAO AND 3rd FERMAT-DAO, *Encyclopedia of Triangle Centers,* available at `http://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16960`

[4] X(16802) = PERSPECTOR OF THESE TRIANGLES: 1st ISODYNAMIC-DAO AND ANTI-EXCENTERS-REFLECTIONS, *Encyclopedia of Triangle Centers,* available at `http://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16802`

[5] X(16940) = PERSPECTOR OF THESE TRIANGLES: 1st LEMOINE-DAO AND ANTI-EULER, *Encyclopedia of Triangle Centers,* available at `http://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16940`

[6] T. O. Dao, *Advanced Plane Geometry*, message 4465, March 20, 2018, available at `https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/4465`

[7] X(3602) = ISOGONAL CONJUGATE OF X(3274), *Encyclopedia of Triangle Centers,* available at `http://faculty.evansville.edu/ck6/encyclopedia/ETCPart3.html#X3602`

[8] X(16271) = CENTER OF THE MORLEY CONIC, *Encyclopedia of Triangle Centers,* available at http://faculty.evansville.edu/ck6/encyclopedia/ETCPart9.html#X16271

[9] T. O. Dao, Advanced Plane Geometry, message 4464, March 20, 2018, available at https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/4464

[10] *Encyclopedia of Triangle Centers,* available at http://faculty.evansville.edu/ck6/encyclopedia/IndexOfTrianglesReferencedInETC.html#F

[11] P. Yiu, On the Fermat lines, *Forum Geom.* 3 (2003) 73–81.