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Deformation of point on circle

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Abstract. In this short note we will introduce generalizations of some theorems in plane geometry.

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Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

In this article we will use main ideas of previous article [3], and introduce generalizations of some theorems in plane geometry. This article can be seen as a continuation of [3].

2. Deformation of point on circle

Consider next non-formal statement $\mathbb{T}$: Any fact which contains some point P on some circle Ω can be deformed into a more general statement, where the point P doesn't lie on the circle Ω. And if we keep moving the point P, to point on Ω, then the general fact will mean a standard one.

3. Examples

Let's use $\mathbb{T}$ to generalize Simson line theorem.

Theorem 3.1 (Simson line theorem)**.** *For any triangle ABC and point P on it's circumcircle, let A', B', C' be feet's of perpendiculars from point P on sides of ABC. Then points A', B', C' lie on same line. This line is the Simson line of P wrt triangle ABC. Moreover we can state that Simson line of point P wrt ABC always intersects with nine-point circle of triangle ABC.*

So if we use $\mathbb{T}$, then we will get next generalization : For any triangle ABC and any point $P \notin (ABC)$, let O be circumcenter of ABC. Reflect line OP wrt angle bisector of angle $\angle APB$, let this line intersect line AB at point C'. Like the same define points A', B' on lines BC, AC respectively. Then points A', B', C' lie on same line. Moreover this line always intersects with nine-point circle of triangle ABC.

Consider next statement (see also [5]) : Let given triangle ABC and circle ω which goes through points B, C. Let M be any point on circle ω. Let π be in-circle of triangle ABC and let line l be tangent line to π which is parallel to line BC. Let tangent lines through point M to circle π intersect line l at two points X_1, X_2. Consider two circles ω_1, ω_2 which are internally tangent to circle ω and also tangent to lines AB, AC. Then circle (MX_1, X_2) is tangent to both circles ω_1, ω_2.

So if we use $\mathbb{T}$, then we will get next fact : Let given triangle ABC and circle ω which goes through points B, C. Let M be any point outside circle ω. Let π be in-circle of triangle ABC and let line l be tangent line to π which is parallel to line BC. Let tangent lines through point M to circle π intersect line l at two points X_1, X_2. Consider two circles ω_1, ω_2 which are internally tangent to circle ω and also tangent to lines AB, AC. Consider circle ψ which is internally tangent to circle ω and also is tangent to lines MX_1, MX_2 Then there exists circle τ which goes through points X_1, X_2 and is tangent to circles ω_1, ψ. (Figure 1)

We finish this article by generalization of next fact [1, Problem 4.7.18]. If we use $\mathbb{T}$ then we can get next statement : Consider points A, B, C, D which lie on same circle ω and C lie between points B, D, point D lie between points C, A. Let $E \notin \omega$ be point which lies outside of ω. Consider circles ω_1, ω_2 which are tangent to line AB at points P, Q, and ω_1, ω_2 are internally tangent to circle ω. Let also circle ω_1 is tangent to line EC, circle ω_2 is tangent to line ED. Consider circle π_1 which is internally tangent to circle ω at point X and is tangent to lines AC, BD. Let lines EC, ED intersect circle ω at points R, L respectively. Consider circle π_2 which is internally tangent to circle ω at point Y and is tangent to lines AR, BL. Then statement says that points P, Q, X, Y lie on same circle. (Figure 3)

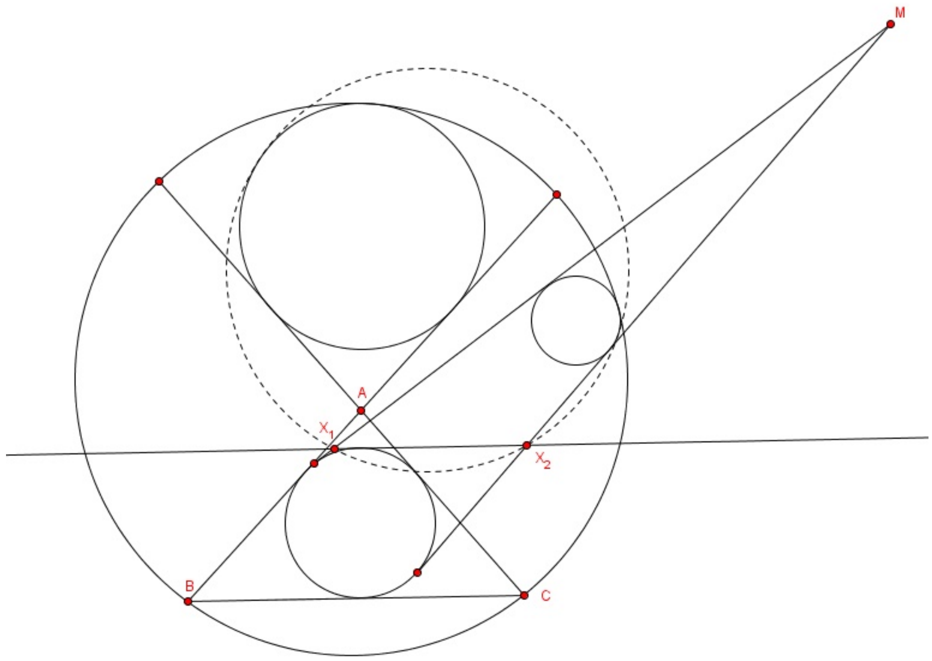

FIGURE 1.

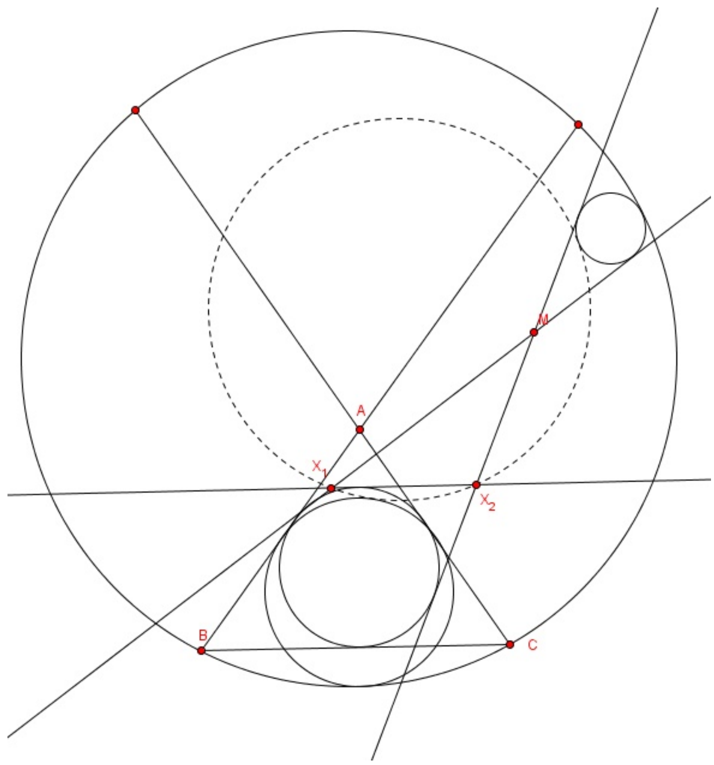

FIGURE 2.

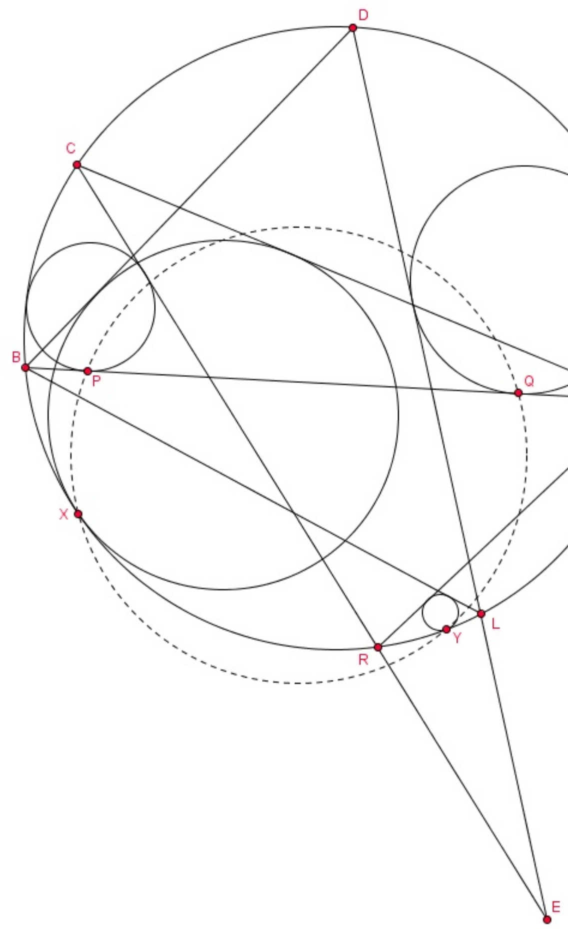

Figure 3.

References

[1] A. V. Akopyan. Geometry in Figures. Createspace, 2011.

[2] *Wikipedia*, https://en.wikipedia.org/wiki/.

[3] *Ways of Predicting Mathematics*, A.Skutin International Journal of Computer Discovered Mathematics, 2016, vol.4, no.1, 1-9. http://www.journal-1.eu/2016-4/Skutin-Predicting-Mathematics-pp.1-9.pdf.

[4] *A concyclic problem*, Nguyen Van Linh http://www.artofproblemsolving.com/community/c6h350775p2172541

[5] *Very nice problem*, http://artofproblemsolving.com/community/c6h275799

[6] *Another Generalization of the Sawayama and Thebaults Theorem*, Dao Thanh Oai International Journal of Computer Discovered Mathematics, 2015, vol.1, no.3, 36-39. http://www.journal-1.eu/2016-3/Dao-Thanh-Oai-Another-Generalization-pp.36-39.pdf.