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Computer Discovered Mathematics: A Note on the Miquel Points

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Abstract. By using the computer program "Discoverer", we give theorems about Miquel associate points.

Abstract. Keywords. Miquel associate point, triangle geometry, remarkable point, computer-discovered mathematics, Euclidean geometry, Discoverer.

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1. Introduction

The computer program "Discoverer", created by Grozdev and Dekov, with the collaboration by Professor Hiroshi Okumura, is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science.

In this paper, by using the "Discoverer", we investigate the Miquel points.

The following theorem is known as the *Miquel theorem*:

Theorem 1.1. *If points A_1, B_1 and C_1 are chosen on the sides BC, CA and AB of triangle ABC, then the circumcircles of triangles AB_1C_1, BC_1A_1 and CA_1B_1 have a point in common.*

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Suppose that P is a point and $\triangle A_1B_1C_1$ is the Cevian triangle of P. Then the Miquel point of circumcircles of triangles AB_1C_1, BC_1A_1 and CA_1B_1 is called the *associated Miquel point of P*.

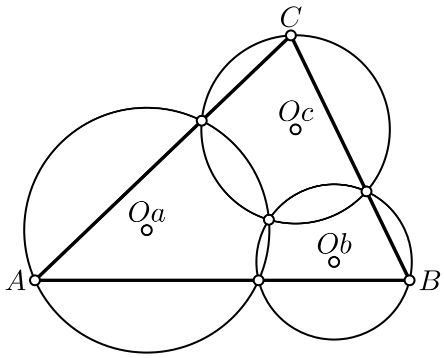

FIGURE 1.

Figure 1 illustrates the Miquel circles. In Figure 1, circles centered at Oa, Ob and Oc are the Miquel circles. They have a common point.

2. PRELIMINARIES

In this section we review some basic facts about barycentric coordinates. We refer the reader to [4], [5], [8], [9], [12], [2], [7], [1].

The labeling of triangle centers follows Kimberling's ETC [6]. For example, X(1) denotes the Incenter, X(2) denotes the Centroid, X(37) is the Grinberg Point, X(75) is the Moses point (Note that in the prototype of the "Discoverer" the Moses point is the X(35)), etc.

The reader may find definitions in [10],[11].

The reference triangle ABC has vertices $A = (1, 0, 0), B(0, 1, 0)$ and $C(0, 0, 1)$. The side lengths of $\triangle ABC$ are denoted by $a = BC, b = CA$ and $c = AB$. A point is an element of $\mathbb{R}^3$, defined up to a proportionality factor, that is,

$$\text{For all } k \in \mathbb{R} - \{0\} : \ P = (u, v, w) \text{ means that } P = (u, v, w) = (ku, kv, kw).$$

Given a point $P(u, v, w)$. Then P is *finite*, if $u + v + w \neq 0$. A finite point P is *normalized*, if $u + v + w = 1$. A finite point could be put in normalized form as follows: $P = (\frac{u}{s}, \frac{v}{s}, \frac{w}{s})$, where $s = u + v + w$.

The equation of the line joining two points with coordinates u_1, v_1, w_1 and u_2, v_2, w_2 is

$$\begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ x & y & z \end{vmatrix} = 0. \tag{2.1}$$

The intersection of two lines $L_1 : p_1x + q_1y + r_1z = 0$ and $L_2 : p_2x + q_2y + r_2z = 0$ is the point

$$(q_1r_2 - q_2r_1, r_1p_2 - r_2p_1, p_1q_2 - p_2q_1) \tag{2.2}$$

The infinite point of a line $L : px + qy + rz = 0$ is the point (f, g, h), where $f = q - r$, $g = r - p$ and $h = p - q$.

The equation of the line through point $P(u, v, w)$ and perpendicular to the line $L : px + qy + rz = 0$ is as follows (The method discovered by Floor van Lamoen):

$$\begin{vmatrix} F & G & H \\ u & v & w \\ x & y & z \end{vmatrix} = 0 \tag{2.3}$$

where $F = S_B g - S_C h$, $G = S_C h - S_A f$, and $H = S_A f - S_B g$.

Given two points $P = (u_1, v_1, w_1)$ and $Q = (u_2, v_2, w_2)$ in normalized barycentric coordinates, the midpoint M of P and Q is as follows:

$$M = \left(\frac{u_1 + u_2}{2}, \frac{v_1 + v_2}{2}, \frac{w_1 + w_2}{2} \right). \tag{2.4}$$

Given two points $P = (u_1, v_1, w_1)$ and $Q = (u_2, v_2, w_2)$ in normalized barycentric coordinates, the reflection R of P in Q is as follows:

$$R = (2u_2 - u_1, 2v_2 - v_1, 2w_2 - w_1). \tag{2.5}$$

3. Theorems

We will give a new proof of the following theorem:

Theorem 3.1. *The Miquel Associate Point M=(uM,vM,wM) of point P=(u,v,w) has barycentric coordinates:*

$$\begin{aligned} uM = a^2(w+u)(u+v)(&b^2u^2w^2 + c^2u^2v^2 - a^2v^2w^2 \\ &+b^2u^2vw + c^2u^2vw - a^2u^2vw \\ &+b^2uv^2w + c^2uv^2w - a^2uv^2w \\ &+b^2uvw^2 + c^2uvw^2 - a^2uvw^2), \\ vM = b^2(u+v)(v+w)(&c^2u^2v^2 + a^2v^2w^2 - b^2u^2w^2 \\ &+c^2u^2vw + a^2u^2vw - b^2u^2vw \\ &+c^2uv^2w + a^2uv^2w - b^2uv^2w \\ &+c^2uvw^2 + a^2uvw^2 - b^2uvw^2), \\ wM = c^2(v+w)(w+u)(&a^2v^2w^2 + b^2u^2w^2 - c^2u^2v^2 \\ &+a^2u^2vw + b^2u^2vw - c^2u^2vw \\ &+a^2uv^2w + b^2uv^2w - c^2uv^2w \\ &+a^2uvw^2 + b^2uvw^2 - c^2uvw^2). \end{aligned}$$

Proof. Let P be a point and $A_1B_1C_1$ be the Cevian triangle of P. Suppose that points A_1, B_1 and C_1 are normalized. We will find the center of circumcircles AB_1C_1 and BC_1A_1 as follows.

By using (2.4) we find the midpoint M_1 of points A and B_1. We have $M_1 = (2u + w, 0, w)$. By using (2.1) and (2.3) we find the equation of the line L_1 through M_1 and perpendicular to the line through points A and B_1.

Then by using (2.4) we find the midpoint M_2 of points A and C_1. By using (2.1) and (2.3) we find the equation of the line through M_2 and perpendicular to the line through points A and C_1.

The intersection of lines L_1 and L_2 is the center of circumcircle of triangle AB_1C_1. We use (2.2) in order to find the center.

Similarly we find the center of circumcircle of triangle BC_1A_1.

Denote by L_o the line through centers of circles and by L_p te perpendicular to it. Denote by Q the intersection point of these lines. Then the associated Miquel point is the reflection of C_1 in Q. We use (2.5) in order to find it. The barycentric coordinates of this points are given in the statement of the theorem. □ □

Table 1 below is given in page 71, [12]:

P	**Miquel associate of P**
Incenter	X(501)
Centroid	Circumcenter
Orthocenter	Orthocenter
Gergonne Point	Incenter
Nagel Point	Bevan Point

TABLE 1

Theorem 3.2. *Table 2 extends Table 1:*

P	**Miquel associate of P**
X(20) de Longchamps Point	X(1498)
X(69) Retrocenter	X(20) de Longchamps Point
X(264) Isotomic Conjugate of the Circumcenter	X(6801)

TABLE 2

Proof. Straightforward verification. □

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