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A Generalization of the Sawayama and Thébault's Theorem

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Abstract. In this note we propose a generalization of the Sawayama and Thébault's theorem associated with a circumconic through two points isogonal conjugate.

Keywords. Euclidean Geometry, Sawayama and Thébault's theorem.

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

Problem 1 ([2] A property of isogonal circumconic). *Let ABC be a triangle, Let P be a point on the plane, and Q be isogonal conjugate of P. Let (w) be the circumconic through two points P, Q. Let (w) meets the circle again at M. Let X be arbitrary on (w). Let XP, XQ meets BC again at D, E respectively. And XM meets the circumcircle again at F. Then (DEF) tangent to the circumcircle*

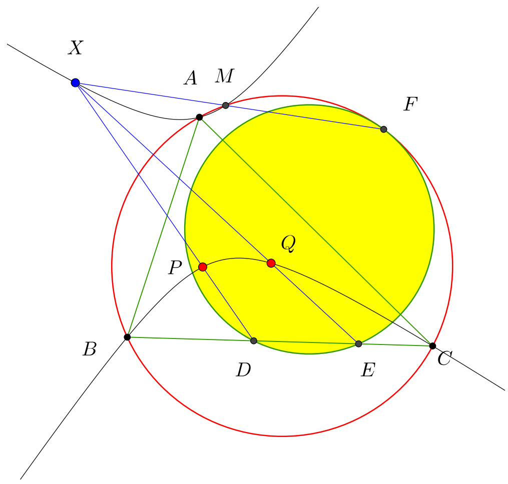

Figure 1. A property of isogonal circumconic

Problem 2 ([3], A generalization of the Sawayama lemma)**.** *Let ABC be a triangle, Let P be a point on the plane, and Q be isogonal conjugate of P. Let E be a point on line BC, DE meets the cirucmcircle at F. Let (ω) be arbitrary circle through two points E, F. Let (Ω) meets BC, PE again at G, H respectively. Let (w) meets AH again at K. Then G, K, Q are collinear.*

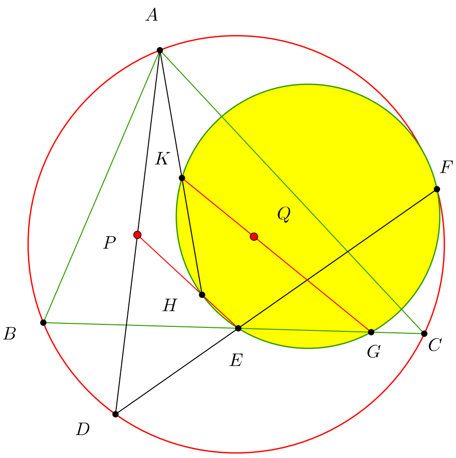

FIGURE 2. A generalization of the Sawayama lemma

Problem 3 ([4])**.** *Let ABC be a triangle, Let P be a point on the plane, Q be the isogonal conjugate of P. Let the line tangent to (BPC) at P meets AQ at Q'. Let (I) be the circumcircle of (PQX'). Let the line AP, AQ, BP, BQ, CP, CQ meet (I) again at A_P, A_Q, B_P, B_Q, C_P, C_Q respectively. Then show that:*

1-A_PA_Q, B_PB_Q, C_PC_Q are concurrent. Let the points of concurrence is D

2-Three antiparallel line of QP respect to triangles APQ, BPQ, CPQ and through A, B, C are concurrent be a point on the circumcircle of ABC

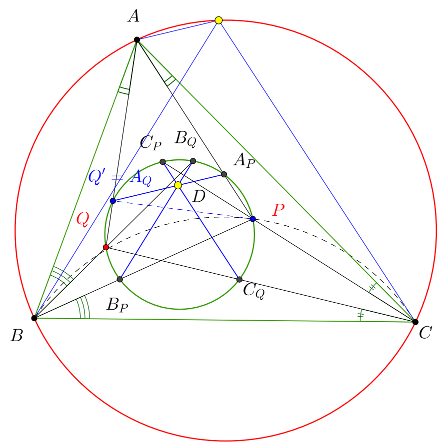

FIGURE 3. Two property of isogonal conjugate

Problem 4 ([4])**.** *Continuing* ***Problem 3****, let the point of concurrence is* D*. Let a line through* D *meets (I) at* M, N*. Let* MP, NQ, MQ, NP *meet* BC *at* M_P, N_Q, M_Q, N_P *respectively. Let* (O_1) *be the circle through* M_P, NQ *and tangent to the circumcircle. Let* (O_2) *be the circle through* M_Q, N_P *and tangent to the circumcircle. Then show that three circle* (O_1)*,* (O_2)*,* (I) *are coaxal. Let* MP, NQ *meet* (O_1) *again at* A_1, A_2*. Let* M_Q, N_P *meet* (O_2) *again at* A_3, A_4 *then show that* A_1, A_2, A_3, A_4 *and* A *are collinear.*

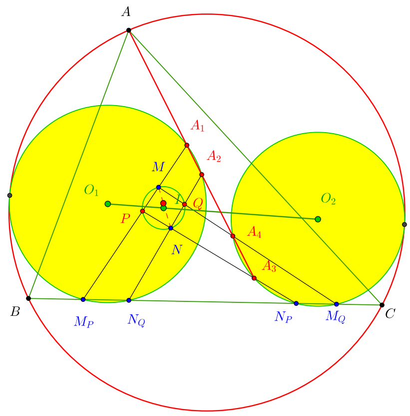

FIGURE 4. A generalization of the Sawayama-Thebault theorem

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