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Computer Discovered Mathematics: A Note on the Gossard Triangle

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Abstract. In this note we use the computer program "Discoverer" in order to study the Gossard Triangle.

Keywords. Gossard triangle, Gossard perspector, triangle geometry, notable point, computer discovered mathematics, Euclidean geometry, "Discoverer".

Mathematics Subject Classification (2010). 51-04, 68T01, 68T99.

1. Introduction

The Gossard perspector (The Zeeman-Gossard Perspector in [8]) is the point X(402) in [8]. The point had appeared in an article by Christopher Zeeman published during 1899 – 1902. In 1016 H. C. Gossard studied the point and the trianle now known as the *Gossard triangle*. In 1998, John Conway noticed that in fact Gossard's triangle is the reflection of triangle ABC in the Gossard Perspector X(402). In 1999 Paul Yiu gave the barycentric coordinates of the Gossard perspector. See [17, Chap. 1, p. 16, Exercise 5], [16, Gossard perspector], [8, Point X(402)], [9].

In this note we use the computer program "Discoverer" [4] in order to study the Gossard Triangle. We use barycentric coordinates. For a survey on barycentric coordinates see [17], [6] [7], [1], [2], [11], [12], [13], [10], [15]. The triangle centers are labeled in accordance with [8].

Geometric Construction of the Gossard perspector is as follow (We use the construction given by Darij Grinberg [3]): Let L be the Euler line of triangle ABC. Denote by X, Y and Z the points of intersection of line L and the sidelines BC, CA

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and AB respectively. Denote by Ma, Mb and Mc the midpoints of segments AX, BY and CZ respectively. The points Ma, Mb and Mc lie on the same line L_1. Then the Gossard perspector X(402) is the intersection point of lines L and L_1.

Figure 1 illustrates the construction.

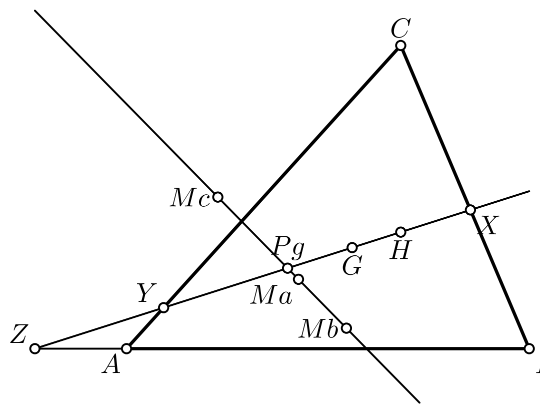

FIGURE 1.

An alternative construction is as follows: Construct the Gossard triangle $AgBgCg$ by the method available in [17, Chap. 1, p. 16, Exercise 5]. Then the Gossard perspector is the perspector of triangles ABC and $AgBgCg$.

Geometric construction of the Gossard triangle is available in Yiu [17, Chap. 1, p. 16, Exercise 5].

We use an alternative approach. First, we construct the Gossard perspector and then we construct the Gossard triangle as the reflection of triangle ABC in the Gossard Perspector.

Note that the Euler lines of triangles $GaGbGc$ and ABC coincide, since the Gossard triangle is the homothetic image of triangle ABC under the homothety with center the Centroid of triangle ABC.

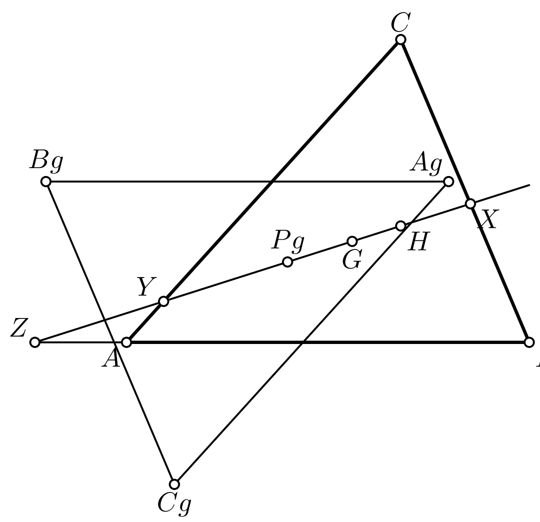

FIGURE 2.

Figure 2 illustrates the Gossard triangle. In Figure 2, Ag is the reflection of A in the Gossard perspector Pg, Bg is the reflection of B in Pg and Cg is the reflection of C in Pg. Then $AgBgCg$ is the Gossard triangle.

2. The Gossard Triangle

2.1. **Barycentric Coordinates.**

Theorem 2.1. *The barycentric coordinates of the Gossard Triangle $AgBgCg$ are as follows:*

$$uAg = (-w+v)^2(2u-w-v),$$
$$vAg = (w+u-2v)(w^2+u^2-v^2-3wu+uv+vw),$$
$$wAg = (u+v-2w)(u^2+v^2-w^2-3uv+vw+wu).$$
$$uBg = (2u-w-v)(-v^2-w^2+u^2+3vw-wu-uv)$$
$$vBg = -(u-w)^2(w+u-2v)$$
$$wBg = (u+v-2w)(u^2+v^2-w^2-3uv+vw+wu)$$
$$uCg = (2u-w-v)(-v^2-w^2+u^2+3vw-wu-uv)$$
$$vCg = (w+u-2v)(w^2+u^2-v^2-3wu+uv+vw)$$
$$wCg = -(u-v)^2(u+v-2w)$$

Proof. Vertex Ag is the reflection of vertex A in the Gossard perspector. We use the barycentric coordinates of the Gossard perspector given by Yiu, See [9]. We use formula [5, §4, (15)] to find Ag. Similarly, we find Bg and Cg. □

2.2. **Notable Points of the Gossard Triangle.**

Theorem 2.2. *The Centroid of the Gossard triang;e $AgBgCg$ is the point X(1651) of triangle ABC.*

Proof. We use barycentric coordinates. We find the reflection of the Centroid in the Gossard perspector. We see that this is point X(1651). □

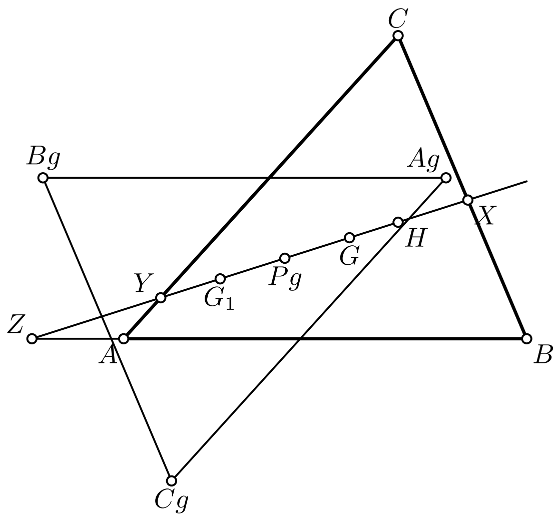

FIGURE 3.

In Figure 3, G_1 is the reflection of the Centroid in the Gossard perspector Pg.

The computer program "Discoverer" easily finds some points of the Gossard triangles. But we see that these points are not available in [8]. See the enclosed Supplementary material. Hence, these points ar new notable points of the triangle.

2.3. **Similitude Centers.**

Problem 2.1. *The External Center of Similitude of the Spieker Circle and the Incircle of the Gossard Triangle is the X(1650)*

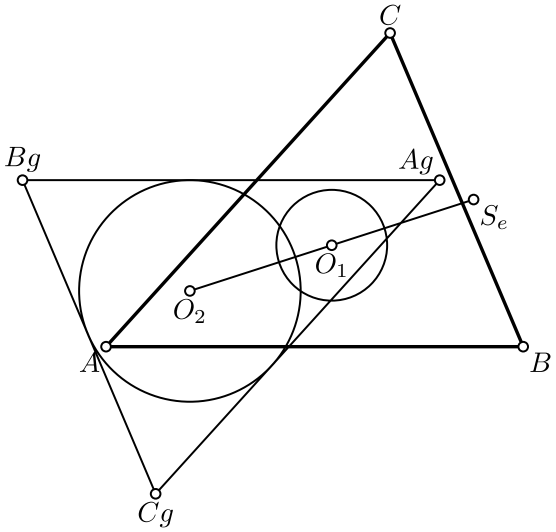

Figure 4.

Figure 4 illustrates Problem 2.1. In Figure 4,O_1 is the center of the Spieker circle of triangle ABC, $GaGbGc$ is the Gossard Triangle, O_2 is the center of the Incircle of triangle $GaGbGc$, point S_e is the External Center of Similitude of circle (O_1) and (O_2).

Problem 2.2. *The Internal Center of Similitude of a named circle of triangle ABC and the corresponding named circle of the Gossard Triangle is the Gossard Perspector X(402).*

Example. The Internal Center of Similitude of the Circumcircle of triangle ABC and the Circumcircle of the Gossard Triangle is the Gossard Perspector X(402).

The reader may find additional problems and examples in the Supplementary Material.

2.4. **Perspectors of the Gossard Triangle.** Clearly, the Gossard perspector is also Homothetic Center. The scale factor of the homothety is -1.

Theorem 2.3. *The Gossard Triangle is homothetic to the Medial Triangle. The Homothetic Center is the point X(1650).*

Theorem 2.4. *The Gossard Triangle is homothetic to the Antimedial Triangle. The Homothetic Center is the point X(4240).*

Supplementary Material

The enclosed supplementary material contains theorems related to the topic.

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