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Mathematics Discovered by Computers: Circles Containing the Parry Point

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Abstract. We present new remarkable circles containing the Parry point. The results are discovered by the computer program "Discoverer".

Keywords. Parry point, remarkable circle, mathematics discovered by computers, triangle geometry, remarkable point, “Discoverer”.

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1. Introduction

The computer program “Discoverer”, created by Grozdev and Dekov, is the first computer program, able easily to discover new theorems in mathematics, and possibly, the first computer program, able easily to discover new knowledge in science. See [3].

In this paper, by using the “Discoverer”, we investigate the circles containing the Parry point. We find 23 different remarkable circles containing the Parry Point. We expect that the majority of results are new, discovered by a computer.

Recall that the Parry Point is the point on the circumcircle of triangle ABC, other than the Steiner point, and on the line connecting the Centroid and the Steiner Point.

The Parry point is labeled X(111) in the Kimberling’s Encyclopedia of Triangle Centers ETC [4]. See also [7, Parry point (triangle)], [6, Parry Point], [5], [1, Parry Point].

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2. THE DERGIADES METHOD

Given three points which are not on the same line. There are a few methods for constructing the equation of the circle through these three points. The methods are almost equivalent. Paul Yiu [9, §15] presents a method due to the Greek mathematician Nikolaos Dergiades. The method is as follows:

The equation of the circle passing through three given points $P_1 = (u_1, v_1, w_1)$, $P_2 = (u_2, v_2, w_2)$ and $P_3 = (u_3, v_3, w_3)$ is as follows:

$$a^2yz + b^2zx + c^2xy - (x + y + z)(px + qy + rz) = 0$$

where

$$p = \frac{D_1}{s_1s_2s_3D},\ q = \frac{D_2}{s_1s_2s_3D},\ r = \frac{D_3}{s_1s_2s_3D},$$

with

$$s_1 = u_1 + v_1 + w_1,\ s_2 = u_2 + v_2 + w_2,\ s_3 = u_3 + v_3 + w_3,\ D = \begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{vmatrix},$$

$$D_1 = \begin{vmatrix} a^2v_1w_1 + b^2w_1u_1 + c^2u_1v_1 & s_1v_1 & s_1w_1 \\ a^2v_2w_2 + b^2w_2u_2 + c^2u_2v_2 & s_2v_2 & s_2w_2 \\ a^2v_3w_3 + b^2w_3u_3 + c^2u_3v_3 & s_3v_3 & s_3w_3 \end{vmatrix},$$

$$D_2 = \begin{vmatrix} s_1u_1 & a^2v_1w_1 + b^2w_1u_1 + c^2u_1v_1 & s_1w_1 \\ s_2u_2 & a^2v_2w_2 + b^2w_2u_2 + c^2u_2v_2 & s_2w_2 \\ s_3u_3 & a^2v_3w_3 + b^2w_3u_3 + c^2u_3v_3 & s_3w_3 \end{vmatrix},$$

$$D_3 = \begin{vmatrix} s_1u_1 & s_1v_1 & a^2v_1w_1 + b^2w_1u_1 + c^2u_1v_1 \\ s_2u_2 & s_2v_2 & a^2v_2w_2 + b^2w_2u_2 + c^2u_2v_2 \\ s_3u_3 & s_3v_3 & a^2v_3w_3 + b^2w_3u_3 + c^2u_3v_3 \end{vmatrix}.$$

3. CIRCLES CONTAINING THE PARRY POINT

Recall that the Parry Point lies on the Circuncircle and the Parry Circle.

Paul Yiu [9, §12] proved that the circles F_+F_-G and GOK also contain the Parry Point.

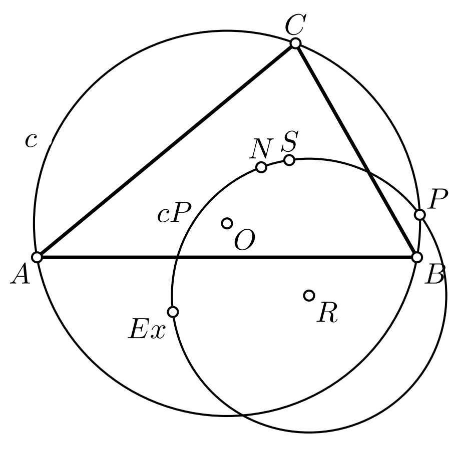

FIGURE 1.

Figure 1 illustrates Theorem 3.1 (9). In Figure 1, N is the Nine-Point Center, S is the Symmedian Point, Ex is the Exeter Point, c is the Circumcircle, cP is the circle passing through points N, S and Ex. Then the Parry Point P lies on circles c and cP.

It is known that the Parry Point lies on the Circle through the Euler Reflection Point, Kiepert Center and Parry Reflection Point. See [1, r507].

The theorem below is discovered by the computer program "Discoverer". The theorem gives 18 additional remarkable circles which contain the Parry Point. Hence, the Parry Point is the point of intersection of 23 different remarkable circles.

Note that Theorem 3.1 extends the result about circles containing the Parry Point, discovered by the prototype of the "Doscoverer" and published in the Journal Computer Generated Euclidean Geometry.

Theorem 3.1. *The Parry Point lies on the following circles:*

(1) *Circumcircle.*
(2) *Parry Circle.*
(3) *Circle through the Centroid, Circumcenter and Symmedian Point.*
(4) *Circle through the Centroid, Inner Fermat Point and Outer Fermat Point.*
(5) *Circle through the Euler Reflection Point, Kiepert Center and Parry Reflection Point.*
(6) *Circle through the Circumcenter, Far-Out Point and Steiner Point.*
(7) *Circle through the Outer Fermat Point, Second Isodynamic Point and Symmedian Point.*
(8) *Circle through the First Isodynamic Point, Inner Fermat Point and Symmedian Point.*
(9) *Circle through the Exeter Point, Nine-Point Center and Symmedian Point.*
(10) *Circle through the Far-Out Point, Kiepert Center and Nine-Point Center.*
(11) *Circle through the Kiepert Center, Schoute Center and Symmedian Point.*
(12) *Circle through the Center of the Brocard Circle, Centroid and Schoute Center.*
(13) *Circle through the Center of the Orthocentroidal Circle, Far-Out Point and Symmedian Point.*
(14) *Circle having as its diameter the line segment connecting the Centroid and Tarry Point.*
(15) *Circle having as its diameter the line segment connecting the Kiepert Center and Tarry Point.*
(16) *Parry Circle of the Fourth Brocard Triangle.*
(17) *Parry Circle of the Pedal Triangle of the Outer Fermat Point.*
(18) *Parry Circle of the Pedal Triangle of the Inner Fermat Point.*
(19) *Parry Circle of the Circum-Symmedial Triangle.*
(20) *Parry Circle of the Circumcevian Triangle of the Far-Out Point.*
(21) *Circumcircle of the Half-Circumcevian Triangle of the Center of the Parry Circle.*
(22) *Orthocentroidal Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Tarry Point.*
(23) *Parry Circle of the Fourth Brocard Triangle of the Neuberg Triangle.*

Proof. (6) The barycentric coordinates of the Circumcenter, Far-Out Point and Steiner Point are available in [4], articles X(3), X(23) and X(99) respectively. We use the Dergiades method for constructing the equation of the circle through the Circumcenter, Far-Out Point and Steiner Point. Then we substitute in the equation of the circle the unknowns x, y, z for the barycentric coordinates of the Parry Point, and we obtain the identity $0 = 0$. Hence the Parry Point lies on the circle.

(14) Note that the barycentric coordinates of the Centroid and the Tarry Point are available in [4], articles X(2) and X(98) respectively. We find the midpoint M of the Centroid and the Tarry Point. We calculate the square of the distance between the midpoint M and the Centroid (we use the normalized coordinates) and denote it by R^2. Then we denote the distance (we use the normalized coordinates) between an arbitrary point $P = (x, y, z)$ and the midpoint M and denote it by dX^2. The equation of the circle having as its diameter the line segment connecting the Centroid and Tarry Point is as follows:

$$dX^2 - R^2 = 0.$$

We substitute in the above equation the unknowns x, y, z for the barycentric coordinates of the Parry Point, and we obtain the identity $0 = 0$. Hence the Parry Point lies on the circle. □

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